Production and Costs

Introduction to Production function, Short Run v/s Long Run and Isoquants

Objectives

After going through this chapter, you shall be able to understand the following concepts.

- Concept of Production and Production Function
- Factors of Production and its types
- Concept of Short Run and Long Run
- Isoquant/Isoproduct Curve and its Properties

Introduction

Production implies creation of goods and services with the sole motive of selling them in the market usually to earn profit. A producer in order to undertake the production process requires some basic prerequisites. These prerequisites are called inputs. The final product produced is called output. Let us understand this with the help of an example. A motor-cycle producer needs steel (raw materials), workers (labour), money (capital), building and avenue (land) for producing motor-cycles. All these requirements are known as ***inputs or factors of production*** and the product (motor-cycle) produced is known as ***output***.

Theory of Production/Producer Theory

As we know that the Theory of Consumer Behaviour focuses on how a rational consumer forms his consumption decisions, how much to consume at what price, how to maiximise his satisfaction, how a consumer attains equilibrium, etc. Similarly, the Theory of Production focuses on how a rational producer forms his production decisions, how much to produce, how to minimise the cost of production and maximise the profit, how a producer attains equilibrium, etc. Producer Theory can be sub-divided into following two theories.

1. Theory of Production- The basic focus of this theory is on how a particular producer makes his production decisions, how much to produce, etc.

2. Theory of Cost- This theory primarily emphasises on the cost of production, how a producer can minimise his cost of production to maximise his profit.

Production Function

The production function of a firm depicts the relationship between the inputs used in the production process and the final output produced. It specifies how much units of different inputs are needed in order to produce the maximum output. Algebraically, a production function can be represented as:

$Q_x = f(L, K)$ (1)

where,

L represents units of labour used (***input one***)

K represents units of capital used (***input two***)

Q_x represents units of output x produced (***output***)

The equation (1) is explained as Q_x units of output x are produced by employing L units of labour and K units of capital by given level of technology. A particular production function is ***associated with is a given level of technology***. If the production technology appreciates (or depreciates), then the maximum level of output increases (or decreases) employing the same input combinations of labour and capital.

Note: *As per the Class-XII syllabus, we will analyse and study two inputs production function, i.e. labour and capital as inputs.*

Factors of Production and Factor Incomes

The inputs used in the production process are called factors of production. These factors are compensated by the producer in exchange of their services and contribution to the output produced. The compensation to the factors in monetary terms is called factor incomes such as, rent, wage, interest and profit. There are four main factors of production namely, land, labour, capital and enterprise.

Land- In economics, land does not merely imply soil. In fact, it broadly refers to all the natural resources, flora and fauna, earth, building, avenues, air, water, minerals, etc. The owner of land (landlord) receives ***rent*** in exchange of his contribution to the production process.

Labour- It includes all physical and mental efforts of workers, employees, managers, etc. A labourer receives ***wages*** in return of his services in the production process.

Capital- It includes money invested in the business, machinery and tools. A money-lender or capital owner receives ***interest*** on the amount of capital contributed by him in the production.

Enterprise- It includes the skills and efforts of the entrepreneur or the owner of the production. An entrepreneur organises the production process by hiring all the services of all the above factors to produce output. He sells the output in the market and remunerates the factor. In exchange of his dare to undertake risk of production, he receives ***profits*** in exchange.

Factors of Production	Owners of Factors	Factor Incomes
Land	Landlord	Rent
Labour	Labourer	Wage
Capital	Moneylender	Interest
Enterprise	Entrepreneur	Profit

Note: *It should be noted that while the factor incomes are the incomes for the factors, on the other hand, it is regarded as cost of production from the producer's (or firm's) point of view.*

Types of factors of Production

The factors of productions are classified as:

1. Variable factors of production
2. Fixed factors of production

Variable Factors of Production- Those factors which can be increased or decreased as per the need to increase or reduce the units of output are called variable factors. The output can be increased or decreased by employing more or less units of variable factors, *ceteris paribus*. Output is a positive function of variable factors, i.e. at zero level of output no (zero) variable factors are employed and as we increase the employment of variable factors, output also increases simultaneously.

Example- Labour is an example of variable factor of production.

Fixed Factors of Production- Those factors which remain constant with the change in the output level are called fixed factors of production. These fixed factors remains constant even at the zero level of output. Let assume that a machine can produce maximum 500 units of output. In this case, fixed factor remains same for all the output units in the range of 0 to 500. That is, the increase in the output units from 0 to 500 can be brought in *without changing the fixed factors*.

Example- Capital such as, building, plant and machinery, etc. are some of the examples of fixed factors of production.

SHORT RUN AND LONG RUN

In economics, time horizon has been divided into two periods- Short run and Long run. There is no explicit base on which such demarcation has been made. That is, the concept of short run and long run may differ from firm to firm on the basis of its nature. For example, for a shoe producing firm, short run implies a period of less than 1 year but for a dam constructing firm, short run may not be less than 8-10 years.

Short Run

In short run, a firm cannot change all its inputs. This implies that the output can be increased (or decreased) by employing more (or less) of variable factor, i.e. labour only. The fixed factors (capital) remain same in the short run. It is generally assumed that in the short run, the firm does not have enough time to change its fixed factors such as, installing a new machinery, thereby the output can only be change by varying the employment of the variable factors. The law which explains this short run concept is called ***Law of Variable Proportions or Returns to Factors***.

A short run production function is expressed as:

$$Q_x = f(L, \overline{K})$$

where,

L represents units of labour used

K represents constant units of capital used

Q_x represents units of output x produced

Long Run

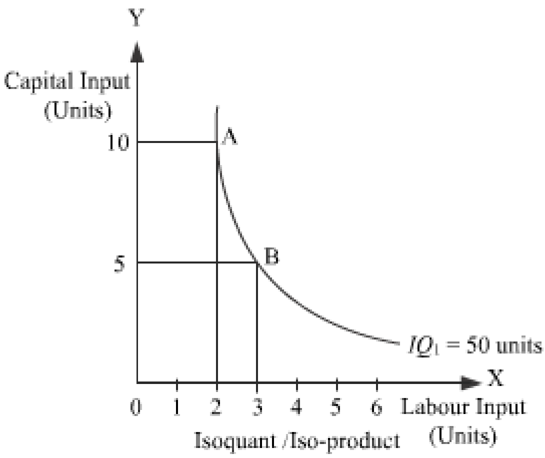
In long run, a firm can change all its inputs. This implies that output can be increased (or decreased) by employing more (or less) of both the inputs- variable and fixed factors. In long run, all inputs (including capital) are variable and can be changed according to the required levels of output. The law which explains this long run concept is called ***Returns to Scale***. The long run production function is expressed as:

$$Q_x = f(L, K)$$

It must be noted that as in the long run, output can be varied by changing both types of inputs, so we can say that in long run all factors of production are variable.

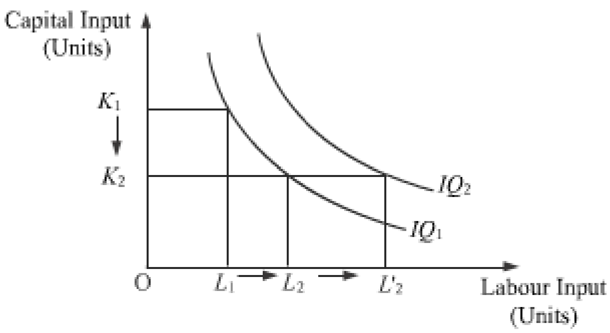
Isoquant/Iso-product Curve/Equal-product Curve

Isoquant refers to the curve that depicts different input combinations which can produce the same level of output. Throughout the isoquant curve, we have the same level of output produced but with different input combinations. The figure below represents IQ_1 as an isoquant curve which is associated with 50 units of output produced.



At point A, the firm is producing 50 units of output by using an input combination of 10 units of capital and 2 units of labour. Similarly, at point B, the firm is producing the same level of output (50 units) but by employing a different input combination of 5 units of capital and 3 units of labour. The firm can choose any point on the IQ_1 to produce 50 units of output depending on the availability of capital and labour units.

1. ***Negative Sloped from Left to Right***- The isoquants are negatively sloped from left to right. This is because if the firm employs more of one input, then it should reduce the employment of another input. As depicted in the figure, if the firm increases the input of labour from OL_2 to OL'_2 *keeping the units of capital same at* OK_2, then the firm is able to produce more units of output and reaches higher isoquant IQ_2. Hence, in order to produce the same level of output (i.e. to remain on the same IQ), the firm must reduce the employment of one input for increasing the employment of other input.



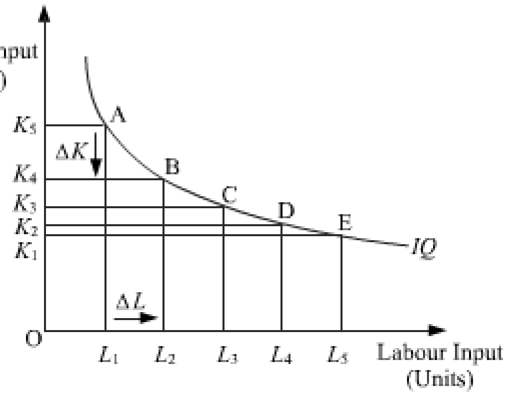
2. ***Isoquants are convex to the origin***- The isoquants are convex to the origin. Convexity of isoquant implies that the two factors are not perfectly substitute to each other. The reason for the convexity of isoquants is diminishing marginal technical rate of substitution. Marginal Rate of Technical Substitution (*MRTS*) is that rate at which one input (say labour) can be substituted for other input (say capital) for producing the same level of output. MRTS is the slope of isoquant curve. It is represented as

$$MRTS = -\frac{\Delta K}{\Delta L}$$

ΔK = change in units of capital

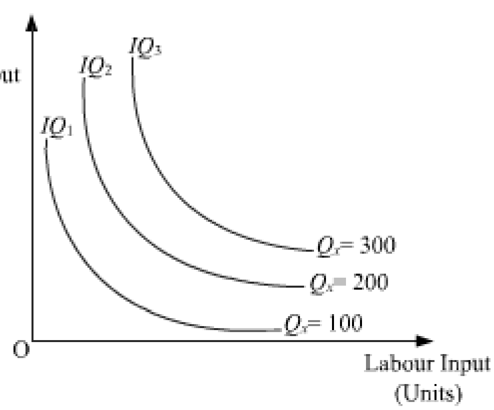
ΔL = change in units of labour

The negative sign implies that an increase in one input cannot be brought without reducing the employment of another input. In the figure below, we can see that as we move along the isoquant curve from left to right i.e. from A to E, the firm is substituting labour for capital at diminishing rate. Thus, moving along the isoquant curve from left to right implies diminishing MRTS, thereby isoquants are convex to the origin.



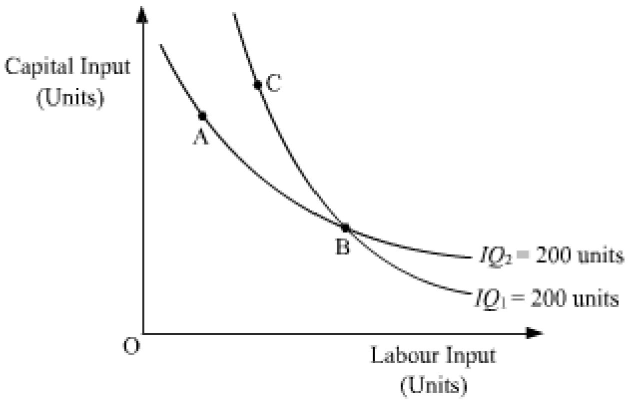
Diminishing Marginal Technical Rate of Substitution

3. ***Higher isoquant curve from origin represents higher output level-*** The isoquant curve which is farther (i.e. higher) from the origin depicts a higher level of output and vice-versa. In the figure, the isoquant curve IQ_3 depicts the production of 300 units of output, while IQ_2 represents the output level of 200 units. IQ_1 depicts the least output level of 100 units in the figure.



4. ***Two isoquants can never intersect each other***- No two isoquants can ever cut each other. All points of a particular isoquant depict the same level of output. If two isoquants happen to intersect each other, then it would imply that at the point of intersection the

output level as depicted by the two different isoquants will be the same. This contradicts the basic characteristic of an isoquant; that higher isoquant represents higher output level. In the figure, IQ_1 represents output level of 100 units, therefore the points A and B represents 100 units of output. The higher isoquant IQ_2 represents output level of 200 units, therefore the points C and B represents 200 units of output. Let assume that the two isoquants cut each other at point B. In this case, the point *B* will depict 100 units of output as it is on IQ_1, but on the other hand, the point B will simultaneously depict 200 units of output as it is also on IQ_2. Thus, it is a contradiction. Hence, we can conclude that no two isoquants can cut each other.



5. ***Isoquants <u>can never</u> be parallel to the vertical and horizontal axis***- The isoquants can neither be parallel to the *x*-axis nor can be parallel to the *y*-axis. This is because the isoquants parallel to the axis implies that for increasing output level, employment of one input can be increased without reducing the input of another factor. This would not lead to the diminishing MRTS. Hence, the isoquants can never be parallel to any of the two axis.

Total Product, Average Product, Marginal Product

Objectives

After going through this chapter, you shall be able to understand the following concepts.

- Concept of Total Product, Average Product and Marginal Product
- Relationship between Total Product, Average Product and Marginal Product

Total Product/Total Physical Product

It is defined as the sum total of output produced by a firm with the all units of inputs- both variable and fixed factors. Total product is also called as Total Physical Product. Algebraically, it is defined as the summation of all the goods and services produced by a firm. In short run, the total product can be increased by employing more units of variable inputs i.e. labour while, in long run as all the factors are variable, so the total product can be increased by employing more units of both the inputs- capital and labour.

$TP = \sum Q_x$

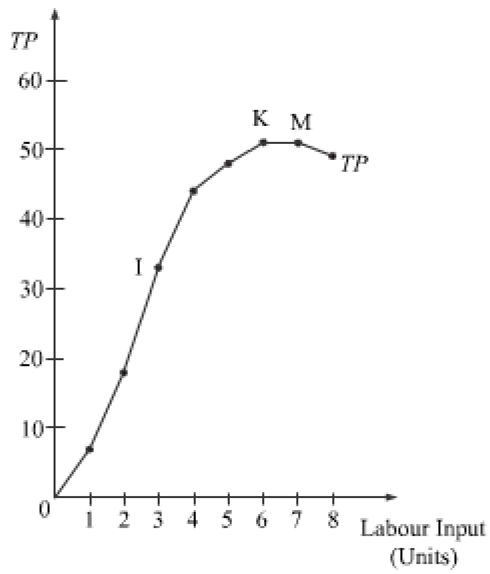
where,

$\sum$ represents summation of all units of output produced

Q_x represents output produced by firm

Let us understand the change in TP curve by considering the schedule and figure below.

Units of Labour	Total Product	Change in the TP curve
0	0	TP rises at an increasing rate till point I
1	7	
2	18	
3	33	
4	44	TP Rises at a decreasing rate from point I to K
5	48	
6	51	TP attains maximum and becomes constant
7	51	
8	49	TP Falls after M



From the figure and the schedule, we can analyse that initially as more and more labour units are employed, the TP curve increases at an increasing rate till point I (corresponding to 2 units of labour units). After point I, the TP curve rises at a decreasing rate till the point K. The point I is also known as ***point of inflexion.*** This is because passing through the point

I, the curvature of the TP curve changes from convex to concave. With successive rise in the labour units, the TP curve continues to rise and attains its maximum point K (corresponding to 6 units of labour) and remains stable till point M. Beyond this point, the TP curve starts falling when more than 8 units of labour are employed.

Average Product

It is defined as output produced by per unit of variable factor (by labour) employed. Algebraically, it is represented as a ratio of Total Product and units of labour employed to produce total output. That is,

$$\text{Average Product} = \frac{\text{Total Product}}{\text{Units of Labour}}$$

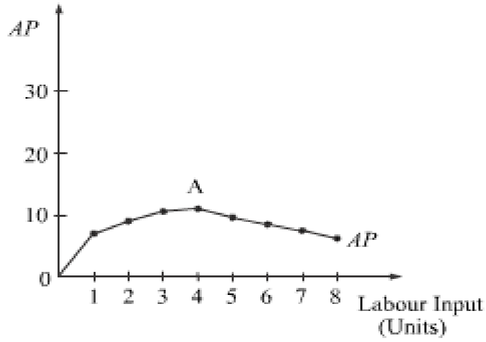
where,

TP represents Total Product

L represents units of labour employed

Let us consider the schedule and figure given below.

Units of Labour	Total Product (TP)	Average Product $AP = \frac{\text{Total Product}}{\text{Units of Labour}}$	Change in the AP curve
0	0	–	AP increases and attains its maximum point at A.
1	7	7	
2	18	9	
3	33	11	
4	44	11	
5	48	9.6	After point A, AP starts falling continue to fall.
6	51	8.5	
7	51	7.4	
8	49	6.1	



Initially, with the increase in the labour units up to 4 units, AP increases and reaches its maximum point A. With successive increase in the labour units beyond 4 units, AP starts falling.

Marginal Product (MP)

It is defined as the additional output due to the employment of an additional unit of labour. In other words, it is the change in the total output brought by employing one additional unit of labour. In other words, MP can be regarded as the contribution by one unit of labour to the total product.

Algebraically, it is defined as the ratio of the change in the total product to the change in the units of labour employed. That is,

$$\text{MP} = \frac{\text{Change in the Total Product}}{\text{Change in the Labour Units}} = \frac{\Delta TP}{\Delta L}$$

or, $MP_n = TP_n - TP_{n-1}$
where,

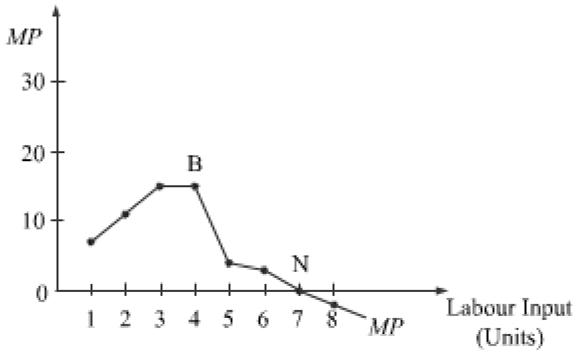
MP_n represents marginal product of n^{th} unit of labour

TP_n = Total product produced by employing n units of labour.

TP_{n-1} = Total product produced by employing $(n - 1)$ unit of labour.

Let us consider the following schedule and figure.

Units of Labour	Total Product (TP)	Marginal Product $MP = \frac{\Delta TP}{\Delta L}$	Change in the MP curve
0	0	–	MP increases an increasing rate and attains its maximum point at B.
1	7	7	
2	18	11	
3	33	15	
4	44	11	After point B, MP curve continues to fall and becomes zero at N.
5	48	4	
6	51	3	
7	51	0	
8	49	–2	After point N, MP curve continues to fall and becomes negative.



Relationship between TP, AP and MP

Let us consolidate all of the above tables together to understand the relationship between TP, AP and MP.

Units of Capital	Units of Labour	TP	AP $AP = \frac{TP}{L}$	MP $MP = \frac{\Delta TP}{\Delta L}$
1	0	0	–	–
1	1	7	7	7
1	2	18	9	11
1	3	33	11	15
1	4	44	11	11
1	5	48	9.6	4
1	6	51	8.5	3
1	7	51	7.4	0
1	8	49	6.1	–2

In the above schedule, we can analyse that the fixed factor (capital) remains same in all the output levels, whereas, more and more labour units are employed to increase output.

TP, AP, MP- Relationship

1. Initially, as more and more units of labour inputs are employed, TP increases at an increasing rate till point I, which is also known as the point of inflexion. After point I, TP increases but at decreasing rate. Simultaneously in the figure (ii), MP attains its maximum point at B (corresponding to the point of inflexion) and thereafter it falls, while AP continues to rise. MP rises at a faster rate than AP.

2. AP attains its maximum point at A. MP continues to fall and cuts AP from its maximum point A, where AP = MP.
3. TP continues to rise but at a decreasing rate, reaches its maximum point at M. Corresponding to the point M, in the figure (ii), MP becomes zero. That is, when TP is maximum, MP becomes zero.
4. As long as TP is positive (and rising), AP and MP are also positive.
5. When TP starts falling, MP becomes negative.
6. Both AP and MP are derived from TP
7. While $AP = \frac{TP}{L}$, $MP = \frac{\Delta TP}{\Delta L}$ or $MP_n = TP_n - TP_{n-1}$
8. Both AP and MP are inverse U-Shaped curve

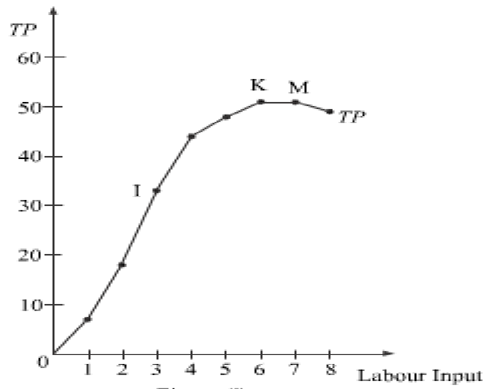


Figure (i)

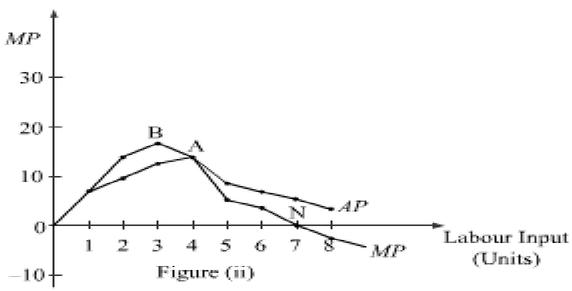
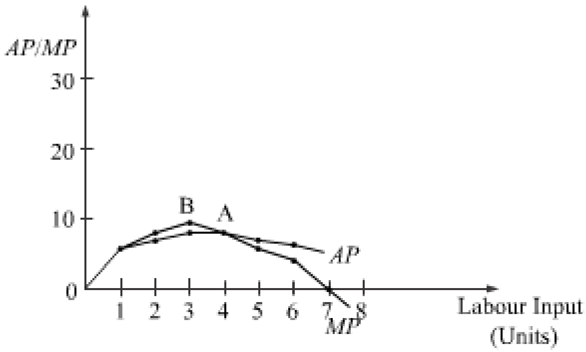


Figure (ii)

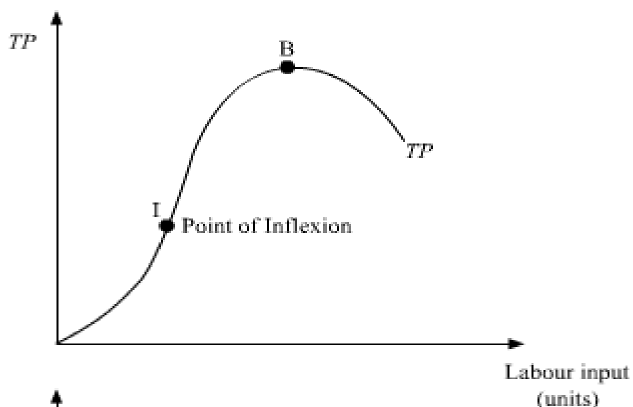
Relationship between AP and MP curve

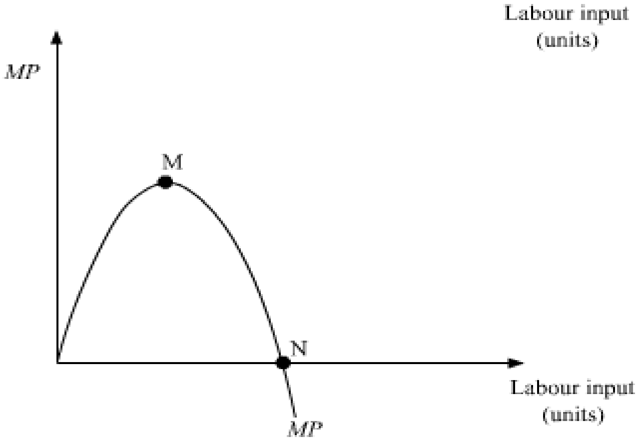
1. Both AP and MP are related to TP i.e. $AP = \frac{TP}{L}$, $MP = \frac{\Delta TP}{\Delta L}$
2. Initially, with rise in the TP curve, AP and MP both rises, but MP rises at a faster rate than AP and attains its maximum point at B.
3. AP continues to rise till point A. MP starts falling after reaching its maximum point B. MP cuts AP at point A, where AP = MP.
4. When AP and MP both fall, MP falls at a faster rate than AP.
5. MP, AP continues to falls, but MP becomes zero (corresponding to the maximum point of TP), thereafter becomes negative. AP remains positive and approaches the *x*-axis, but never touches the axis.



Relationship between TP and MP

1. TP is defined as the total output produced by the total number of inputs employed. MP is defined as an additional to the total product by an additional unit of input.
2. MP is derived from TP as: $MP = \frac{\Delta TP}{\Delta L}$ or $MP_n = TP_n - TP_{n-1}$
3. When TP increases at an increasing rate, MP also increases at an increasing rate.
4. When TP increases at decreasing rate, MP attains its maximum point and starts falling.
5. When TP reaches its maximum point, MP becomes zero.
6. When TP starts falling, MP becomes negative and continues to fall.





Numerical Examples

1. When MP is given and TP and AP are to be calculated.

L	MP_L	To be Calculated	
		$TP_n = TP_{n-1} + MP_n$	$AP_L = \frac{TP_L}{L}$
1	5	5	$\frac{5}{1} = 5$
2	7	5 + 7 =12	$\frac{12}{2} = 6$
3	9	12 + 9 = 21	$\frac{21}{3} = 7$
4	7	21 + 7 = 28	$\frac{28}{4} = 7$
5	5	28 + 5 = 33	$\frac{33}{5} = 6.6$
6	3	33 + 3 = 36	$\frac{36}{6} = 6$

2. When TP is given and AP and MP are to be calculated.

L	TP_L	To be Calculated	
		$AP_L = \frac{TP_L}{L}$	$MP = TP_n - TP_{n-1}$
0	0	–	–
1	20	20	20 (20 – 0)
2	36	18	16 (36 – 20)
3	51	17	15 (51 – 36)
4	44	11	– 7 (44 – 51)
5	25	5	– 19 (25 – 44)

3. When AP is given and TP and MP are to be calculated.

L	AP_L	To be Calculated	
		$TP_L = AP \times L$	$MP = TP_n - TP_{n-1}$
1	20	20 × 1 = 20	20
2	30	30 × 2 = 60	60 – 20 = 40
3	40	40 × 3 = 120	120 – 60 = 60
4	40.25	40.25 × 4 = 161	161 – 120 = 41
5	40	40 × 5 = 200	200 – 161 = 39
6	30.5	30.5 × 6 = 183	183 – 200 = –17

Law Variable Proportions and Law of Diminishing Marginal Product - Returns to a Factor

Objectives

After going through this chapter, you shall be able to understand the following concepts.

- Concept of Law of Variable Proportions, Law of Diminishing Marginal Returns, Assumptions and Exceptions
- Reasons for Increasing Returns, Constant Returns and Decreasing Returns
- Viable and Non-Viable Stages of Production

Introduction

As we have learnt that with successive increase in the labour inputs, initially TP increases at an increasing rate, then becomes constant and afterwards decreases. The change in TP is only because of the change in the variable factor, i.e. labour, keeping the fixed factor (capital) constant. The law that explains this behaviour (of TP) in the short run is known as Law of Variable Proportions.

Law of Variable Proportions

The Law of Variable Proportions states that if more and more of variable factor (labour) is combined with the same quantity of fixed factor (capital), then initially the total product will increase but gradually after a point, the total product will become smaller and smaller.

Law of Diminishing Marginal Product

According to the Law of Diminishing Marginal Product, if the employment of variable factor is kept on increasing along with the constant level of the fixed factor, then finally a point will be reached whereafter, the marginal product of the variable factor will start falling and after this point the marginal product of any additional variable factor can be zero and even be negative.

Note: ***Both the above laws explain the same concept but from two different perspectives. While the Law of Variable Proportions is based on the Total Product of a firm, whereas, the Law of Diminishing Marginal Product is based on the marginal product of the variable factor.***

Assumptions of Law of Variable Proportions

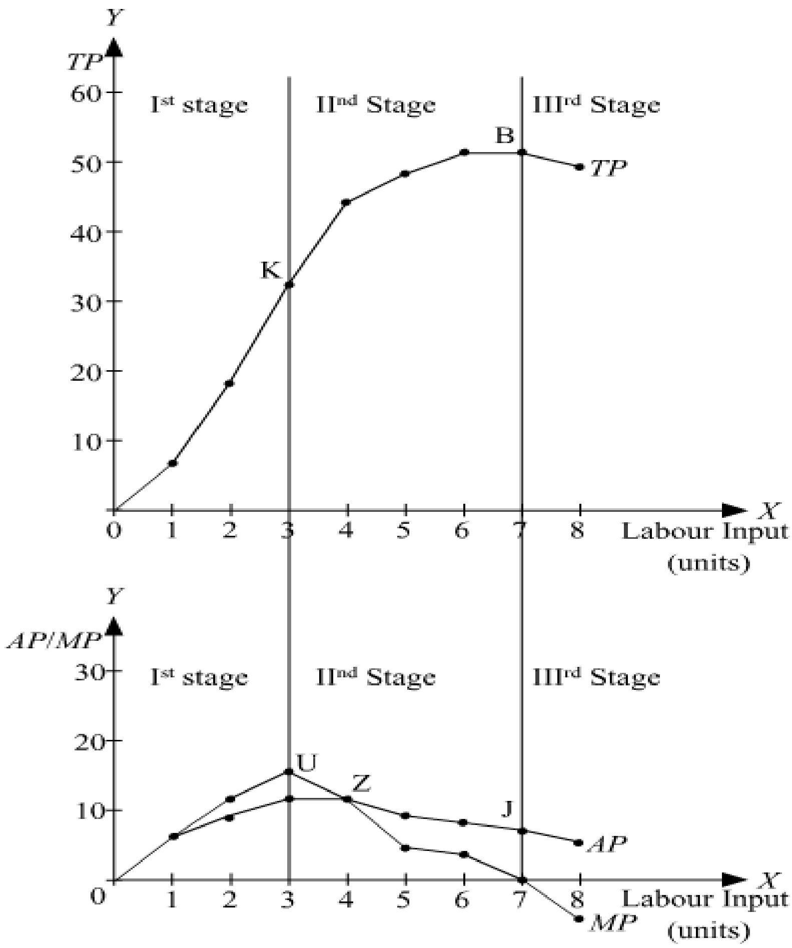
1. Technology level remains constant
2. The units of variable factor are homogeneous. This implies that all the labour units are equally productive.

3. One of the input must be fixed and it is because of the limited availability of this factor that diminishing returns exists.
4. No change in the input prices i.e. wages (price of labour) and interests (price of capital) remains the same.
5. It is assumed that proportions in which the factors are combined can be varied to produce the desired output level. For, example, if for producing 10 units of output 3 units of labour and 1 unit of capital are employed, then to produce 15 units of output 5 units of labour and 1 unit of capital can be combined. The input ratio in the first instance is 3:1, while in the second instance is 5:1. The law does not apply when all factors are to be varied proportionately (i.e. in long run).

Explanation of Law of Variable Proportions/Law of Diminishing Marginal Returns

In order to understand the laws and how they operate let us analyse the following schedule and the figure.

Units of Capital	Units of Labour	TP	AP $AP = \frac{\text{Total Product}}{\text{Units of Labour}}$	MP $MP = \frac{\Delta TP}{\Delta L}$
1	0	0	0	–
1	1	7	7	7
1	2	18	9	11
1	3	33	11	15
1	4	44	11	11
1	5	48	9.6	4
1	6	51	8.5	3
1	7	51	7.4	0
1	8	49	6.1	–2



The whole production phase can be distinguished into three different production stages.

Ist Stage: Increasing Returns to a Factor

This stages starts from the origin point O and continues till the point of inflexion (K) on the TP curve. During this phase, TP increases at an increasing rate and is also accompanied by rising MP curve (in figure ii). The MP curve attains its maximum point (U) corresponding to the point of inflexion. Throughout this stage, AP continues to rise.

Reasons for Increasing Returns to a Factor

1. *Underutilisation of the fixed factor*- In the first stage of production, there are not enough labour units to fully utilise the fixed factor. Therefore, the firm can increase its output just by combining more and more of labour inputs with the fixed factor, thereby; the output of the additional unit of labour (i.e. MP) tends to rise.
2. *Division of labour*- The increase in the labour input enables the division of labour, which further increases the efficiency and productivity of the labour.

3. *Specialisation of labour*- Due to the division of labour, specialisation of individual labour unit increases, which in turn raises the overall efficiency and productivity. Consequently, the MP curve rises and TP curve continues to rise.

Important Note: It must be noted that the first stage of production is also known as ***non-economic zone*** as a rational producer will not operate profitably in this stage. This is because throughout this stage, the marginal product of labour is rising which implies that production (or total product) can be increased by employing more units of labour. Thus, the rational producer will attempt to increase the total product by increasing the labour units and move forward to the second stage of production.

IInd Stage: **Diminishing Returns to a Factor**

This stage starts from point K and continues till point B on the TP curve. During this stage, the TP increases but at a decreasing rate and attains its maximum point at B, where it remains constant. On the other hand (in the figure ii), the MP curve continues to fall and cuts AP from its maximum point Z, where MP equals AP. When TP attains its maximum point, corresponding to it, MP becomes zero. AP, in this stage initially rises, attains its maximum point at Z and thereafter starts falling.

Reasons for Decreasing Returns to a Factor

1. *Fuller utilisation of fixed factor*- In this stage, the fixed factor is utilised to its maximum level as more and more of labour inputs are employed.
2. *Imperfect substitutability between labour and capital*- The variable factors are imperfect substitute for the fixed factor. Therefore, the firm cannot substitute labour for capital and as a result diminishing returns takes place.
3. *Optimum Proportion/ Ideal Factor Ratio*- The optimum proportion (or ideal factor ratio) is a fixed ratio in which the labour and capital inputs are employed. These factors will be the most efficient if they are employed as per the optimum proportion. If this proportion is disturbed (by combining more of labour inputs to the fixed units of capital), then the efficiency of the factors will fall, thereby leading to the diminishing returns to the factor.

Important Note: This stage is also known as ***economic zone*** as any rational producer will like to (and always) operate in this zone. This is because in this stage, the fixed factors are fully utilised and the efficiency of the factors is the maximum.

IIIrd Stage: Negative Returns to a Factor

This stage begins from the point B on the TP curve. Throughout this point, TP curve is falling and MP curve is negative. Simultaneously, the AP curve continues to fall and approaches the *x*-axis (but does not touch it). Like the first stage, this stage is also known as ***non-economic zone*** as any rational producer would not operate in this zone. This is because the addition to the total output by the additional labour unit (i.e. marginal product) is negative. This implies that employing more labour would not contribute anything to the total product but will add to cost of the production in form of additional wage. Hence, the cost of the additional labour input is greater than the benefit of employing it.

Reasons for Negative Returns to a Factor

1. *Over utilisation of the fixed factors*- In the third stage of production, the variable factor is in excessive relative to the fixed factors. This leads to the over utilisation of the fixed factor, thereby negative returns to a factor sets-in.
2. *Negative Marginal Product*- Throughout this stage the TP curve is continuously falling, consequently, the additional product by the additional unit of labour becomes negative. This implies that in this stage of production, the cost of employing labour is substantially higher than its contribution to the total product.
3. *Problem of Management*- With the increased number of labour units employed, it becomes hard for the management of the firm to efficiently manage them. Thus, due to the mismanagement and lack of responsibility, inefficiency is infused in the system.

Summary of Law of Variable Proportions

Stages	Stage Name	TP	AP	MP	Domain
I	Increasing Returns to a factor	TP increases at an increasing rate till K	AP continues to increases	MP also rises but at faster rate than AP and becomes maximum.	From O to point K
II	Diminishing Returns to a factor	Increases at a decreasing rate and attains maximum	AP reaches its maximum	MP falls and cuts AP from its maximum point and finally becomes zero.	From K to point B
III	Negative Returns to a factor	TP starts falling	AP continues to fall	MP continues to fall and become negative.	From B onwards

Exception to the Law of Variable Proportions/ Postponement of the Law

The Law of Variable Proportions fails to operate in the following situations.

1. *Appreciation of level of technology*: If the available technology level appreciates, then the productivity of labour will increase with the same level of fixed factor, consequently, the Law of Variable Proportions may become inoperative.
2. *Discovery of substitutes of fixed factors:* This implies that if all the factors becomes variable due to the substitutability between the factors, then the output can be changed by changing the factors, thereby the law of diminishing marginal product would not set-in.

3. *Heterogeneous labour units*: The law operates under the assumption of homogeneous labour units. That is, all labour units are equally productive and efficient. But in case, if the labour units are heterogeneously (differently) productive, then the law fails to operate.
4. *Efficiency of fixed factor*: If initially the fixed factors were utilised with lesser efficiency, then at the later phases the decrease in the marginal product can be postponed by increasing the efficiency with which the fixed factors are utilised.

Applicability of Law of Variable Proportion

Nevertheless of all the above mentioned limitations, this law is universally applicable to any spheres of production. This law is applicable primarily in production areas such as, agricultural and industrial sectors. This law helps us to understand the reason behind falling marginal product in agricultural sector when more and more labourers are employed on the same piece of land. Secondly, it helps the firms to draft their short run production plans accordingly.

Viable and Feasible Stage of Production- 'In which stage of production does a rational producer operate?'

The viable and feasible stage of production refers to the economically efficient phase of production. As we have observed above that out of the three phases of production, the first and third stages of production are called non-economic zone and a rational producer does not operate in these stages. While in the first stage, the MP of the labour is positive which implies that production can be increased by employing more and more of labour units with the same quantity of fixed factor. Secondly, in this stage, the labour units are not fully utilising the available fixed factors. Thus, we can conclude that any producer would attempt to raise production by increasing the labour input and will reach the second stage of production. On the other hand, in the third stage, the MP of labour is negative which implies that production can be increased by reducing or shedding the numbers of labour. Therefore, the *stage second is the only viable and economically feasible* stage. In this stage, MP, AP and TP all are positive and the available fixed factors are fully utilised. This is the most efficient stage where the productivity of the factors is the maximum, so a rational producer would always like to operate in this stage.

Return to Scale

Objectives

After going through this chapter, you shall be able to understand the following concepts.

- Concept of Law of Returns to Scale
- Reasons for Increasing Returns, Constant Returns and Decreasing Returns to Scale
- Difference between Returns to Factor and Returns to Scale

Introduction

We know that the Law of Variable Proportions (or Law of Diminishing Marginal Product) is applicable only in the short run with the one factor remaining constant and other factor as variable. In the following chapter, we are going to understand the law that is applicable in the long run, when all the factors are variable. The law that explains the long run production concept is Law of Returns to Scale.

Law of Returns to Scale

According to the Law of Returns to Scale, if all the factor inputs are increased in the same proportion, then consequently the output will increase; but this increase may be at increasing, constant or at decreasing rate.

Based on the increase in the output, there exists following three aspects of Law of Returns to Scale.

1. Increasing Return to Scale (IRS)
2. Constant Returns to Scale (CRS)
3. Decreasing Returns to Scale (DRS)

Increasing Returns to Scale

It holds when a proportional increase in all the factors of production leads to an increase in the output by more than the proportion.

Example- When both labour and capital inputs are increased by n times and the resultant increased in the output level is more than n times; we say that the production function exhibits IRS.

Algebraically

Let $Q_x = f(L, K)$

where,

Q_x represents Quantity of output produced

L represents 'L' units of labour used

K represents 'K' units of capital used

If both the inputs are increased by 'n' times, then the new output, Q'_x will be
$Q'_x = f(nL, nK)$
Now, if $\boldsymbol{f(nL, nK) > n \cdot f(L, K)}$, then the production function shows IRS.

Constant Returns to Scale

It holds when a proportional increase in all the factors of production leads to an equal proportional increase in the output.

Example- If both labour and capital input are increased by 10% and the resultant output also increases by 10%, then we say that the production function exhibits CRS.

Symbolically, it exists when
$Q_x = n.\ Q'_x$

That is, $\boldsymbol{f(nL, nK) = n\ .\ f(L, K)}$, then the production function shows CRS.

Decreasing Returns to Scale

It holds when a proportional increase in all the factors of production leads to less than the proportional increase in the output.

Example- If both labour and capital are increased by 10% but the resultant increase in the output is less than 10%, then we say that the production function exhibits DRS.

Symbolically, it exists when
$Q_x < n.\ Q'_x$

That is, $\boldsymbol{f(nL, nK) < n\ .\ f(L, K)}$, then the production function shows DRS.

Causes for Increasing Returns to Scale

Increasing Returns to Scale implicitly implies reduction in the cost of production. The reducing costs may be attributed to the economies of scale. The economies of scale refer to those benefits that accrue to a particular firm due to increase its size and expansion of its business overtime. The economies of scale can be further categorised into the following two types of economies.

1. Internal Economies
2. External Economies

Internal Economies

These are those cost reduction efforts which are created by an individual firm itself with the increase in the scale of production. Such economies are only limited and specific to a particular firm as they exist due to expansion in the firm's size, technological appreciation, employment of well-trained managers and workers, increase in the specialisation of the employees and scale of operation.

Types of Internal Economies

1. ***Technical Economies***- These economies arise because due to the use of better machinery and improved technology of production. As a firm expands overtime, it earns sufficient funds to install costly and advanced machinery and also to employ sophisticated technology. These are more efficient and enhance productivity, thereby lower the overall cost of production.
2. ***Managerial Economies***- These economies arise due to qualitative and elaborated management, which can be afforded only by the large firms. These large firms can employ highly skilled managers and can also incur huge and large scale advertisement. This enables more availability of scarce finance to undertake production at large scale.
3. ***Financial Economies***- As a large and renowned firm has a good goodwill in the market, so it can issue new shares and debentures in the primary share market. In addition, it can also get cheaper bank loans. These benefits cannot be enjoyed by their smaller counterparts.
4. ***Marketing Economies***- Due to bulk demand, a big firm purchases raw materials and capital goods at lower and cheaper rates. It can also set-up its own R&D (research and development) department and wholesale distributor. This will not only reduces the cost of production but also ensures a continuous revival and qualitative improvement of output of the firm.
5. ***Risk and Survival Economies***- Usually the large firms are less prone to shocks and risks. This is because the large firms often engage in diversification of their business, market and production techniques. This helps them to minimise the risk involved with the change in the business environment.

2. **External Economies**- Unlike internal economies, these economies are not specific to any particular firm. In fact, these economies are shared by a number of firms within an industry. These economies are exogenous to a particular firm and occur with the increase in the scale of production (of all the firms) of an industry.

Types of External Economies

1. ***Economies of Concentration***- When many firms of a particular industry establish themselves at one place, then they enjoy the benefits due to their concentration. Some of these benefits are cheap availability of raw materials, transport facilities, skilled labour, development of new inventions etc. All these benefits directly or indirectly contribute to the simultaneous growth of all the firms.
2. ***Economies of Information***- All the firms can join hands together and pool their finance together to invest in R & D department. Further, the cost of exchanging information and ideas within themselves also reduces. This facilitates in quality improvement of product of the industry as a whole.
3. ***Economies of Disintegration***- Concentration of firms help the firms to specialise and efficiently produce sub-products. These individual sub-products are used together for the production of a single final product. For example, in a car manufacturing industry some firms specialise in the production of wheels, some in the body of the car while some in the

engine of the car. This leads to specialisation and efficient production of the cars by the industry as a whole.

Causes for Decreasing Returns to Scale

Decreasing Returns to Scale implies increasing cost of production. The increase in the cost of production is a result of diseconomies of scale. Such diseconomies exist when a firm grows and expands beyond its optimum (maximum) capacity. In other words, a firm faces diseconomies of scale whenever it engages in overproduction. The diseconomies of scale can be categorised into following two categories.

1. Internal Diseconomies
2. External Diseconomies

Internal Diseconomies are *internal* and are *specific* to a particular firm. Such diseconomies obstruct a firm to produce efficiently at higher levels of output. These may exist due to managerial obstacles, financial obstacles, and technical obstacles.

External Diseconomies are *exogenous* and *external* to an individual firm. These diseconomies exist in form of increase in unit cost of production due to the expansion and growth of the industry. These diseconomies have spill-over effect on other firms as well.

Difference between Returns to Factor and Returns to Scale

Basis of Differences	Returns to a Factor	Returns to Scale
1. Time Period	This law is applicable only in the ***short run***, when output can be varied only by changing variable factor of production.	This law is applicable only in ***long run***, when output can be varied by varying all factors of production.
2. Production	According to this law, ***level*** of production can be changed.	According this law, ***scale*** of production can be changed.
3. Different Stages of Production	There exist three stages i.e. Increasing Returns, Diminishing Returns and Negative Returns to a Factor.	There exist three stages i.e. Increasing Returns, Constant Returns and Decreasing Returns to Scale.
4. Factor Ratio	In this law, we assume that factor ratio changes.	In this law, we assume that factor ratio remains constant.

5. Nature of Production Function	This law refers to variable proportions type of production function.	This law refers to constant proportions type of production function.

Concepts of Costs, Shapes of Cost Curve in Short Run and Long Run

Objectives

After going through this chapter, you shall be able to understand the following concepts.

- Concept of Cost of Production
- Short Run and Long Run Costs, Types of Costs
- Relationship between various costs

Introduction to Cost of Production

The cost of production refers to the expenditures incurred or payments made by a firm to various factors of production (such as, land, labour, capital and entrepreneur) and also non-factors of production (such as, raw materials, etc.) In other words, cost of production can be understood as the payments made in exchange of services or contribution of the factors and non-factors to the production process.

Cost Function

The algebraic form that depicts the functional relationship between cost of production and output is called cost function. It is expressed as:

$C = f(Q_x)$

where,
C represents cost of production
Q_x represents units of output '*x*' produced

A cost function depicts the least cost combination of inputs associated to different levels of output.

Cost in Different Time Horizons

According to the different time horizons, the cost of production can be categorised into the following two different categories.

1. Short run costs
2. Long run costs

Short Run Costs

Those costs which are incurred in the short run for producing output are called short run costs. In short run, some factors remains fixed such as, capital while some are variable such as, labour. Those costs which are incurred on the fixed factors are called ***Fixed Costs*** or ***Total Fixed Costs*** (TFC hereafter) and those costs which are incurred on the variable factors are called ***Variable Costs*** or ***Total Variable Costs*** (TVC hereafter). Due to the presence of constant fixed costs, therefore, any producer attempts to minimise the short run cost of production by minimising the variable costs.

The short run costs are the summation of both fixed as well as variable costs. Algebraically:

TC or STC = TFC + TVC

where,

TC or STC represents Total Cost or Short Run Total Cost

TFC represents Total Fixed Cost

TVC represents Total Variable Cost

Types of Cost in Short Run

There are mainly three types of costs that are incurred by a firm in the short run.

1. Total Cost
2. Average Cost
3. Marginal Cost

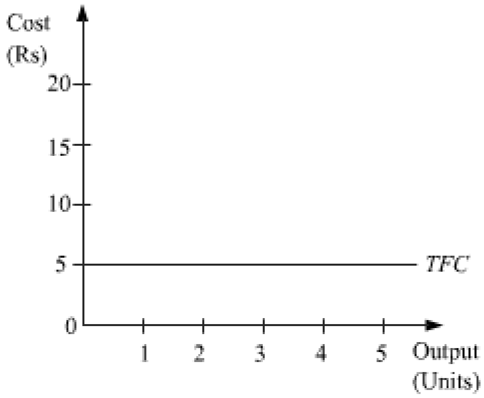
Total Cost

It refers to the total cost of production that is incurred by a firm in the short run to carry out the production of goods and services. It is the aggregate of expenditure incurred on fixed factors as well as variable factors. Therefore, the total cost can be segmented into following two parts namely- TFC and TVC. That is,

TC = TFC + TVC

Total Fixed Costs- These refer to those costs which are incurred by a firm in order to acquire the services of the fixed factors for production. In short run, fixed factors cannot be varied and accordingly the fixed costs remain same (constant) throughout all output levels. Cost of machinery, buildings, depreciation on fixed assets, etc. are some of the examples of the fixed costs. These are also known as ***overhead costs***.

Output (Units)	**TFC** (in Rupees)
0	5
1	5
2	5
3	5
4	5
5	5

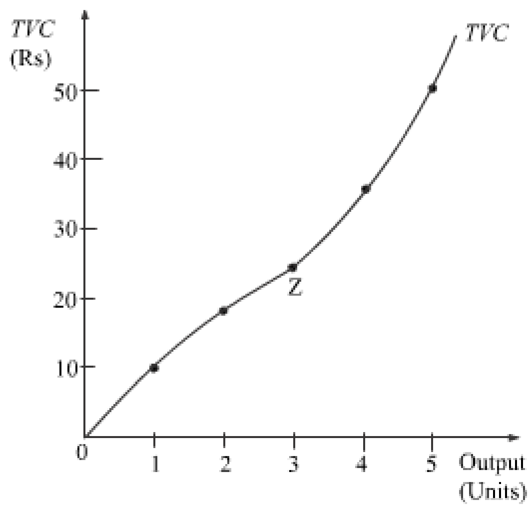


Analysing the TFC schedule and the figure, we can say that TFC remains constant irrespective of the output levels. Therefore in short run TFC is a horizontal curve parallel to *x*-axis.

Total Variable Costs- These refers to those costs which are incurred by a firm on the variable inputs for production. The variable costs are positive function of output i.e. as output increases, variable costs also increases and vice-versa. That is, as more and more units of labour are employed to produce higher units of output, accordingly the variable costs rises. These costs are also known as ***Prime Costs*** or ***Direct Costs*** and include expenses such as:

1. Wages of labour
2. Fuel expenses
3. Costs of raw materials

Output (Units)	**TVC** (in Rupees)
0	0
1	10
2	18
3	24
4	36
5	50



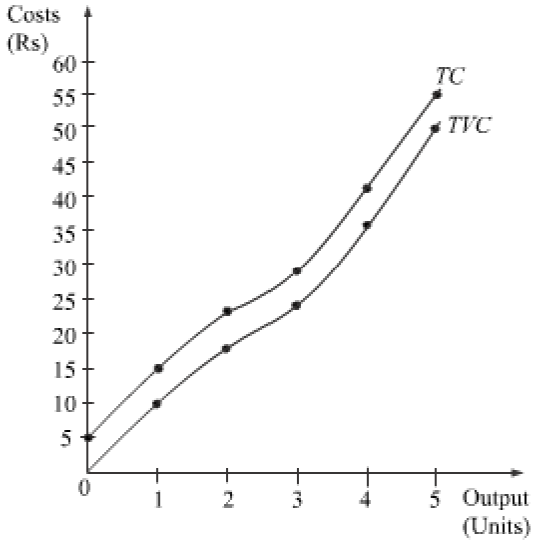
Shape of TVC- Inverse S-shaped

We can know that TVC varies as output varies. At zero level of output, TVC is also zero as the firm is not producing anything. With the initial rise in the output level, TVC rises at decreasing rate, whereas, in the later stages of rise in the output level, the TVC rises at increasing rate. The reason for the inverse-S shape of TVC can be attributed to the Law of Variable Proportions. In the initial stage of production, due to the increasing returns to labour, marginal product of labour rises which implies reducing costs and hence, TVC increases at a decreasing rate till point Z. After point Z, due to the diminishing returns to labour, marginal product of labour falls, hence, TVC rises at an increasing rate.

Difference between TVC and TFC

Basis of Differences	TVC	TFC
1. Definition	These costs are incurred on the variable inputs and includes expenses such as, expenses on raw materials, wages, fuel expenses, etc.	These costs are incurred on the fixed inputs and includes expenses such as, cost of machinery, cost of building, etc.
2. With respect to changes in the output level	TVC changes with the changes in the level of output produced.	TFC remains constant irrespective of level of output.
3. Graphical representation	TVC is an inverse S-shaped curve	TFC is parallel to *x*-axis.
4. Avoidable	TVC can be avoidable by firm if the firm is not producing any unit of output	TFC cannot be avoidable even if the firm is not producing any unit of output.
5. Rigidity	TVC is flexible and can be change in the short run.	TFC is fixed and cannot be change in the short run.

Superimposing TVC on TFC graph we get TC.



Output (in units)	TFC (in Rs)	TVC (in Rs)	TC = TVC + TFC (in Rs)
0	5	0	5
1	5	10	15
2	5	18	23
3	5	24	29
4	5	36	41
5	5	50	55

Based on the given figure and the schedule, we can say that the difference between TC and TVC is due to TFC. The vertical distance between TC and TVC is equal to TFC. *As TFC is fixed, so the shape of TC is inherited from shape of TVC.* The logic behind the shape of TC is (same as it is for the shape of TVC) Law of Variable Proportions.

Average Cost

Average Cost is defined as per unit cost of producing output. It is derived by dividing total cost by quantity of output produced. That is,

$$AC = \frac{TC}{Q}$$

AC is the sum total of Average Fixed Cost and Average Variable Cost. That is,

$AC = AFC + AVC$

where,

AC represents Average Cost

AFC represents Average Fixed Cost

AVC represents Average Variable Cost

Q represents units of output produced

Average Fixed Cost

It is defined as the fixed cost per unit of output produced. It is derived by dividing the Total Fixed Cost by quantity of output produced. That is,

$$AFC = \frac{TFC}{Q}$$

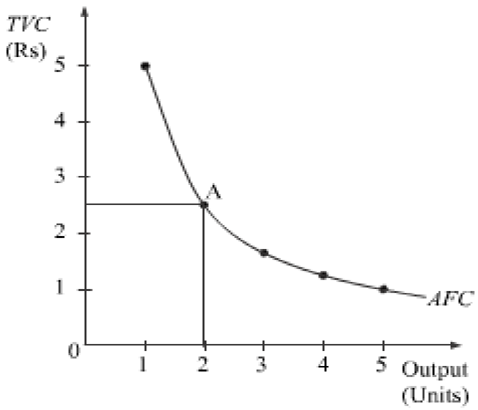
where,

TFC represents Total Fixed Cost

Q represents units of output produced

Schedule of Average Fixed Cost

Output (units)	TFC	AFC (Rs) $AFC = \frac{TFC}{Q}$
0	5	∞
1	5	5
2	5	2.5
3	5	1.66
4	5	1.25
5	5	1



Shape of AFC- Rectangular Hyperbola Shape

We can see in the figure that AFC is downward sloping rectangular hyperbola. This is because at any point on AFC curve, if AFC is multiplied by corresponding unit of output, then we get TFC. For example, at point A in the above figure, AFC is Rs 2.5 and corresponding output produced is 2 units, hence, TFC is Rs 5 (i.e. Rs 2.5 × 2)

Average Variable Cost

It is defined as the variable cost per unit of output produced. It is derived by dividing the Total Variable Cost by quantity of output produced. That is,

$$AVC = \frac{TVC}{Q}$$

where,

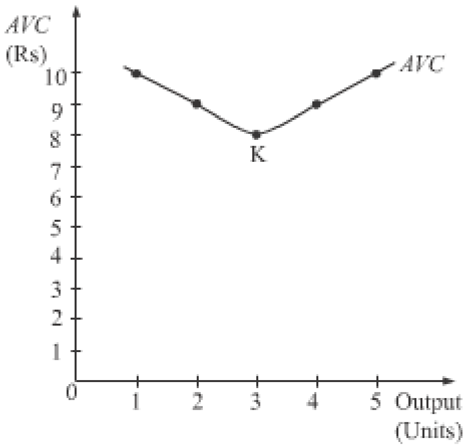
AVC represents Average Variable Cost

TVC represents Total Variable Cost

Q represents units of output produced

Schedule of *AVC*		
Output (units)	*TVC*	*AVC* (Rs) $AVC = \frac{TVC}{Q}$
0	0	∞
1	10	10
2	18	9
3	24	8
4	36	9

5	50	10



Shape of AVC: U-Shaped Curve

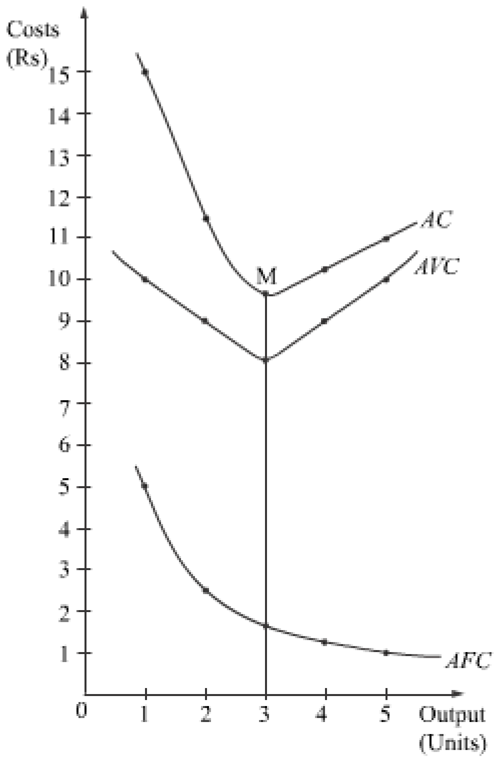
AVC is a U-shaped curve. That is, as output increases, the AVC curve falls and reaches its minimum point 'K' and then rises up. The reason behind the U-shape of AVC curve is the Law of Variable Proportions. In the initial stage of production, due to the increasing returns to labour, the variable cost falls i.e. marginal product is rising, so AVC curve falls. But at point 'K', owing to the constant returns to labour, AVC curve tends to stabilise. Beyond this point, as the firm is employing additional labour (with the constant unit of fixed input), the MP of the additional labour falls implying decreasing returns to the labour. This result in rising costs, thereby, AVC starts rising up. Therefore AVC is a 'U-shaped' curve.

Average Cost

Average Cost curve is vertical summation of Average Fixed Cost and Average Variable Cost. Superimposing the AFC curve and AVC curve in one figure, we get Average Cost.

Schedule of AC, AVC, AFC			
Output (units)	***AFC*** (in Rs)	***AVC*** (in Rs)	***AC = AFC + AVC*** (in Rs)

0	∞	0	∞
1	5	10	15
2	2.5	9	11.5
3	1.66	8	9.66
4	1.25	9	10.25
5	1	10	11



From the graph we can analyse that AFC curve is continuously falling, whereas, the AVC curve initially falls, then stabilises and thereafter increases beyond three units of output. Vertically summing up AVC and AFC we derive AC curve.

U-Shaped Short Run Average Cost Curve

Short run Average Cost curve is a U-shaped curve which implies that initially AC falls then reaches its minimum point and rises afterwards. The reason behind AC being U-shaped is the Law of Variable Proportions. According to this law, in the initial stages of production, MP of labour (MP_L) rises as the fixed factors are not fully utilised. The rising MP_L implies more output can be produced by per additional unit of labour; consequently cost of labour per unit of output falls. Therefore, in the initial stage of production AVC falls and therefore AC also falls.

AC continues to fall till its minimum point M and becomes stable due to the constant returns to labour. This point indicates the best combination of capital and labour, as the capital is optimally utilised by labour units. Beyond this point M, AVC starts rising due to over utilisation of capital. Therefore, MP_L starts falling, which implies higher variable costs and AVC curve starts rising. But the rise in AVC curve offsets (or is greater than) the fall in AFC, which pushes the AC curve to rise.

Therefore, it can be summed up that at the onset, falling AVC along with falling AFC leads AC curve to fall. But later on, as AFC becomes smaller and smaller, the rising AVC leads AC curve to rise. Thus, we can say that AC curve is U-shaped curve because AVC curve is also a U-shaped curve.

Important Note: *It must be noted that the distance between AC curve and AVC curve tends to diminish, but the two curves would never meet. This is because AFC is a rectangular hyperbola, which has a property that it would never touch the x-axis. Thus, at higher level of output, AFC becomes smaller and smaller, consequently, AC curve and AVC curve tends to converge.*

Marginal Cost

It is defined as the additional cost to the Total Cost, which is incurred for producing one more unit of output. It can be calculated by either of the following two formulas.

1. $MC_n = TC_n - TC_{n-1}$

where,

MC_n represents Marginal Cost of producing '*n*' units of output

TC_n represents Total Cost of producing '*n*' units of output

TC_{n-1} represents Total Cost of producing '*n* – 1' units

2. $MC = \frac{\Delta TC}{\Delta Q} = \frac{\Delta TVC}{\Delta Q}$

ΔTC represents change in Total Cost i.e. $TC_n - TC_{n-1}$

ΔTVC represents change in Total Variable Cost i.e. $TVC_n - TVC_{n-1}$

ΔQ represents change in output

$\Delta Q = Q_n - Q_{n-1}$

It can also be said that the sum total of Marginal Cost is Total Variable Cost. This is because in the short run one factor remains fixed, so any additional increase in the cost should be on the cost on the employing an additional variable factor (or labour). Therefore, the sum total of all the marginal cost of producing '*n*' units of output is called TVC of producing n^{th} unit of output. That is,

$\Sigma MC = TVC$

***Example*- 1**

Output (units)	***MC*** (in Rs)
1	10
2	8
3	6
4	4
5	4
	$\Sigma MC = 10 + 8 + 6 + 4 + 4 =$ Rs 32 $= TVC$

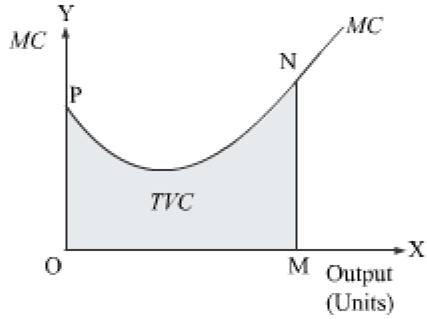
***Example*-2**

Output (units) **(Column 1)**	***TVC*** (in Rs) **(Column 2)**	***TFC*** (in Rs) **(Column 3)**	***TC*** = ***TVC*** + ***TFC*** (in Rs) **(Column 4 = Col. 2 + Col. 3)**	***MC*** (in Rs) **(5)**
0	0	5	5	-
1	10	5	15	10
2	18	5	23	8
3	24	5	29	6
4	36	5	41	12
5	**50**	5	55	14
				ΣMC = **50** = *TVC*

Therefore, we can say that the sum total of Marginal Cost of producing 5 units of output is Rs 50, which is the TVC of producing 5th unit of output.

Geometrically, the area (shaded region) under MC curve corresponding to different output levels indicates the TVC of that output level. That is,

$$TVC = \text{area of } MNOP = \sum_{i=1}^{5} MC$$

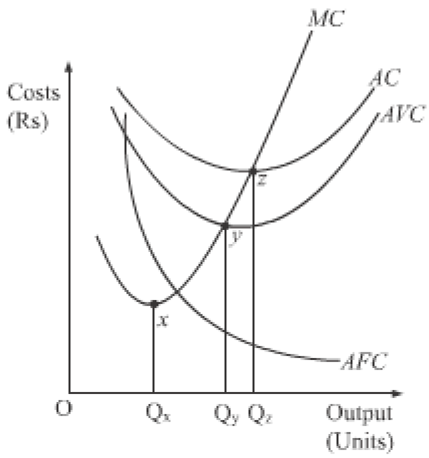


Relationship between AFC, AVC, AC and MC Curves

Preparing AFC, AVC, AC and MC curves in one diagram will help us in understanding the relationship between them more easily.

Schedule of AC, AVC, AFC and MC

Output (units)	*AFC* (in Rs)	*AVC* (in Rs)	*AC* = *AFC* + *AVC* (in Rs)	*MC* (in Rs)
0	∞	0	∞	-
1	5	10	15	10
2	2.5	9	11.5	8
3	1.66	8	9.66	6
4	1.25	9	10.25	12
5	1	10	11	14



Relationship between AC and MC

1. When AC is falling, MC falls at a faster rate; and MC remains below AC curve.
2. When AC is rising, MC rises at a faster rate; and MC remains above AC curve.
3. When AC is at its minimum point, (z), MC is equal to AC.
4. MC curve cuts AC curve at its minimum point.
5. AC and MC are both U-shaped curve reflecting the Law of Variable Proportions.
6. While AC includes both variable as well as fixed cost, whereas, MC includes only the variable cost.
7. AC and MC are both derived from TC as:

i.e. $$AC = \frac{TC}{Q}$$

and $$MC = \frac{\Delta TC}{\Delta Q}$$

Relationship between AVC and MC

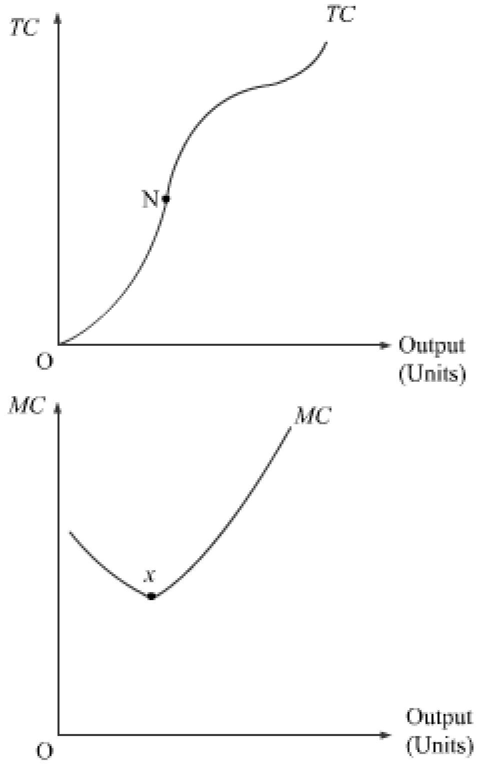
1. When AVC is falling, MC falls at a faster rate and stays below AVC curve.
2. When AVC is rising, MC rises at a faster rate and remains above AVC curve.
3. When AVC is at minimum point (y), MC is equal to AVC.
4. MC curve cuts AVC curve at its minimum point.
5. The minimum point of MC curve (x) will always lie left to the minimum point of AVC curve (y).
6. AVC and MC both are derived from TVC.

$$AVC = \frac{TVC}{Q}$$

$$MC = \frac{\Delta TC}{\Delta Q} = \frac{\Delta TVC}{\Delta Q}$$

Relationship between TC and MC curve

Output (units)	***TC* (in Rs)**	***MC* (in Rs)**
0	5	-
1	15	10
2	23	8
3	29	6
4	41	12
5	55	14
		$\Sigma MC = 50 = TVC$



1. When TC curve rises at decreasing rate, MC curve is decreasing.
2. When TC curve rises at an increasing rate, MC curve rises.
3. When TC curve stops rising at decreasing rate i.e. at point '*N*' (after which it starts rising at an increasing point), MC curve reaches its minimum point (*x*).
4. While TC consists of both variable as well as of fixed costs (i.e. TVC and TFC), MC consists of only variable costs.
5. TC curve is an inverse S-shaped curve, whereas, MC curve is inverted U-shaped curve.
6. MC is derived from TC as:

$$MC = \frac{\Delta TC}{\Delta Q}$$

i.e. MC is the slope of TC curve.

Long Run Costs

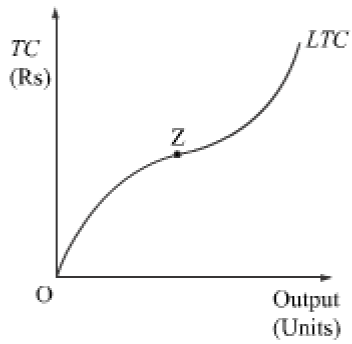
The costs which are incurred in the long run for producing output are called long run costs. As in the long run, all factors of production are variable, i.e. the output level can be varied by varying both capital as well as labour, so there is *no concept of fixed costs in long run and all costs are variable costs*. In the long run, a firm has sufficient time to change the scale of production by choosing the optimum factor ratio, i.e. the best combination of capital and labour, install new machinery and to employ advanced technology, so as to minimise the cost of production.

Similar to the short run, in long run we have mainly three important costs.

1. Long Run Total Cost Curve (LTC)
2. Long Run Average Cost (LAC)
3. Long Run Marginal Cost (LMC)

Long Run Total Cost Curve (LTC)

As in the long run all factors are variable, so unlike short run, LTC consists of only variable costs and no fixed costs. The shape of LTC is similar to that of the short run total cost.



Important Note: *It should be noted that the LTC curve starts from the origin. This is because at zero level of output, the firm is not employing any input.*

Initially, the LTC curve rises at diminishing rate due to increasing returns to scale. This implies that when the firm increases all factor inputs by certain proportion, then the increase in output is more than the proportionate increase in factor inputs. The LTC curve continues to rise at a diminishing rate till the point '*Z*' after which it starts increasing at an increasing rate due to diminishing returns to scale. It is due to the fact that the diseconomies associated with the large scale production are more than the economies of scale and as a result, the LTC starts rising at an increasing rate.

Long run Average Cost (LAC)

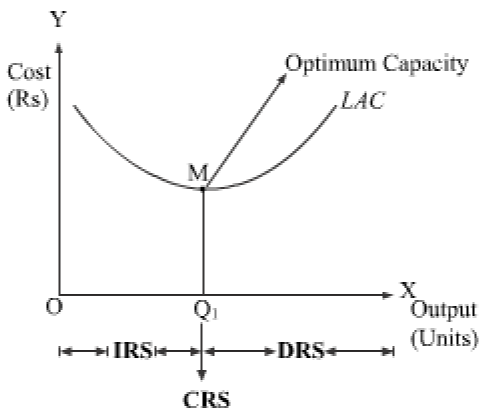
LAC shows the cost of producing per unit of output when all factors are variable. It is important to note that the LAC curve shows the minimum cost per unit of producing each level of output in presence of all factors being variable.

$$LAC = \frac{LTC}{Q}$$

Shape of LAC Curve: U-Shaped Curve

LAC curve is also a U-shaped curve, as is the short run AC curve. The U-shape of LAC can be attributed to the ***Law of Returns to Scale***.

It is argued that a firm generally experiences increasing returns to scale (IRS) during the initial period of production in long run, then followed by constant returns to scale (CRS) and lastly by decreasing returns to scale (DRS), consequently LAC is U-shaped curve. Due to IRS, as output increases, LAC falls, due to the economies of scale. The falling LAC then experiences CRS at Q_1 level of output which is also known as the *optimum capacity*, beyond Q_1 level of output, the firm experiences diseconomies of scale and as a result, the cost of production rises.



Long run Marginal Cost (LMC)

LMC is the addition cost to the long run total cost, which is incurred to produce one more unit of output when all factors are variable. LMC curve is also U-shaped curve. It can be calculated by either of the following two formulas.

$LMC_n = LTC_n - LTC_{n-1}$

where,

LMC_n represents Long run Marginal Cost of producing 'n' units of output

LTC_n represents Long run Total Cost of producing 'n' units of output

LTC_{n-1} represents Long run Total Cost of producing 'n – 1' units

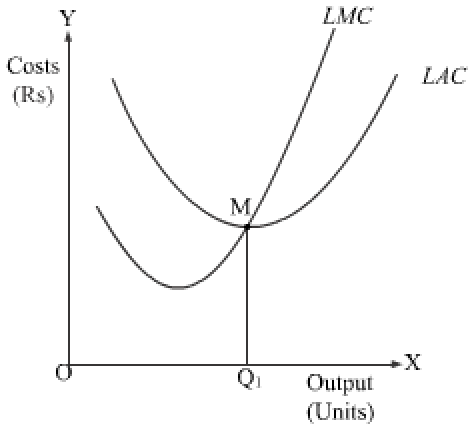
$$LMC = \frac{\Delta LTC}{\Delta Q}$$

ΔLTC represents change in Long run Total Cost i.e. $LTC_n - LTC_{n-1}$

ΔQ represents change in output

$\Delta Q = Q_n - Q_{n-1}$

Relationship between LMC and LAC



1. When the LAC curve is falling, the LMC curve is also falling but at a faster rate and remains below the LAC curve.
2. When the LAC curve is rising, the LMC curve is also rising but at a faster rate and remains above the LAC curve.
3. When the LAC curve is constant (i.e. at the minimum point '*M*'), the LMC is equal to LAC.
4. The LMC curve cuts the LAC curve at its minimum point.
5. Both LMC and LAC are derived from LTC as:

i.e. $LAC = \frac{LTC}{Q}$

and $LMC = \frac{\Delta LTC}{\Delta Q}$